Receding-Horizon Energy-Maximising Optimal Control of Wave Energy Systems Based on Moments

Nicolás Faedo, Yerai Peña-Sanchez, and John V. Ringwood

Abstract—In this study, we address the issue of real-time energy-maximising control for wave energy converters (WECs), by proposing a receding-horizon optimal control framework based on the concept of a moment. This approach is achieved by extending the so-called moment-based framework, recently published in the WEC literature, to effectively solve the associated optimal control problem within a finite time-horizon, allowing for real-time performance, and a straightforward inclusion of the wave excitation force estimation and forecasting requirements, which are intrinsic to the wave energy control application. We present a case study, based on a CorPower-like device, subject to both state and input constraints. We show that the proposed strategy can perform almost identically to the ideal performance case, where full knowledge of \( F_w \) over the time-horizon is assumed available. Moreover, a sensitivity analysis is provided, addressing the impact of wave excitation force estimation and forecasting errors in the computation of the moment-based control input. Two main conclusions can be drawn from this analysis: Forecasting mismatch has a negligible impact on the overall performance of the strategy, while potential differences arising from estimating \( F_w \) in particular, phase errors, can substantially impact total energy absorption.

Index Terms—Wave energy, WEC, receding-horizon, energy-maximising control, optimal control, moment-domain.

I. INTRODUCTION

OPTIMAL ENERGY extraction for wave energy converters (WECs) has the capabilities of reducing the levelised cost of energy extracted from ocean waves, hence greatly helping in the roadmap towards the commercialisation of WEC technologies [1]. Such an objective is virtually always formulated in terms of an optimal control problem (OCP). Not only has this real-time OCP to be solved efficiently in computational terms, but energy-maximisation can only be achieved by having full (instantaneous and future) knowledge of the wave excitation force \( F_w \), i.e. the force experienced by the WEC due to incoming waves. Unfortunately, for the WEC case (i.e. a moving body), \( F_w \) is, in general, immeasurable. Consequently, unknown-input state estimation strategies are virtually always required to provide instantaneous values of \( F_w \) (see [2]). Based on these estimates, a number of forecasting techniques have been proposed to predict future wave excitation force within a certain time interval [3]. Naturally, the uncertainty of such a prediction increases with longer time horizons, offering a relatively precise prediction (in realistic sea state conditions) for no more than \( 3 \sim 10 \) s.

Motivated by both the real-time requirements, and the intrinsic estimation and forecasting needs associated with this OCP, receding-horizon approaches to WEC control became popular over the last decade, where a number of solutions emerged, stemming from the basic principles of model predictive control (MPC) [4]. Nevertheless, MPC formulations, which are typically based on zero- or first-order hold discretisations (i.e. compactly supported basis functions), have (at least) two main drawbacks. Firstly, the objective function employed departs from pure energy-maximisation, given that the problem (discretised as described above) is inherently non-convex, so that the control objective has to be modified to render the OCP tractable. Secondly, the computational requirements of MPC-based strategies can render this approach unsuitable for real-time control of WECs [4]. Spectral [5] and pseudospectral [6], [7] approaches have been proposed, aiming to solve these issues, in which both the OCP and system variables are discretised using sets of global basis functions. [6] effectively provides a computationally efficient solution, but the OCP is modified (similarly to MPC) to solution existence. In contrast, [5] and [7] provide strategies that both consider a purely energy-maximising objective function, and are appealing from a computational perspective, but the existence of solutions to the OCPs proposed is neither guaranteed nor discussed, so that it is not clear under which conditions these OCPs admit an energy-maximising solution, compromising the feasibility of the corresponding pseudospectral approaches.

Recently, a novel energy-maximising control strategy has been presented in [8], [9]. This formalism is based on the system-theoretic concept of a moment [10] and maps the original energy-maximising OCP into a concave quadratic program (QP), systematically guaranteeing a unique global solution for the original energy-maximising control objective, subject to both state and input constraints. Though [8], [9] accomplishes the energy-maximising objective, subject to state and input constraints, the mathematical formalism assumes a sufficiently long time interval, where \( F_w \) is known into the future, to solve the OCP, which is limiting in terms of real-time implementation.

This paper directly addresses the issue of real-time energy-maximising control of wave energy converters, including the estimation and forecasting needs associated with the WEC control problem, as detailed in the following. Motivated by the appealing features of the moment-based approach, we propose...
an extension of the strategy proposed in [8], [9] (which is limited in terms of real-time implementation), by introducing a finite (receding) horizon moment-based OCP framework. In particular, this paper provides the following contributions:

- Unlike [8], [9], which requires full knowledge of the wave excitation force $F_w$ throughout a sufficiently large time-interval, the representation of $F_w$ in the moment-domain is adapted to finite (short) prediction horizons by an appropriate mathematical definition. To that end, and inspired by the technique employed in [5], we propose the use of a family of apodisation mappings [11], which effectively alleviate the effects of ‘ignoring’ the assumptions on $F_w$, required in [8], [9].

- Based on the representation of $F_w$ proposed in the item immediately above, a moment-based receding-horizon real-time controller is proposed. We show that the proposed framework retains the intrinsic computational efficiency, and the uniqueness of the corresponding solution, facilitated through the parameterisation of the corresponding OCP using moments.

- Using the unknown-input estimation strategy presented in [12], and the autoregressive (AR) model of [3], we present performance results, in terms of energy-capture, for a full-scale state-of-the-art heaving CorPower\footnote{The reader is referred to [13] for further detail on the CorPower device.} device, subject to both state and input constraints.

- Finally, we provide a sensitivity analysis, addressing the impact of estimation and forecasting errors on the computation of the moment-based optimal control input and, hence, on total energy absorption. Using the results of this sensitivity analysis, we give a set of recommendations, related to the design of estimation and forecasting techniques for WEC control applications.

The remainder of this paper is organised as follows. Section II introduces the basics behind moment-based theory, while Section III introduces the energy-maximising problem for WECs, using a receding-horizon approach. Section IV details the receding-horizon moment-based control formulation proposed in this paper, while Section V discusses an application case, including a corresponding sensitivity analysis. Finally, Section VI articulates the main conclusions of this study.

### A. Notation and Preliminaries

Standard notation is considered throughout this study. $\mathbb{R}^+$ ($\mathbb{R}^-$) denotes the set of non-negative (non-positive) real numbers. $\mathbb{C}^0$ denotes the set of pure-imaginary complex numbers, and $\mathbb{C}_{<0}$ denotes the set of complex numbers with negative real part. The symbol $0$ stands for any zero element, dimensioned according to the context. The notation $\mathbb{N}_q$ indicates the set of all positive natural numbers up to $q$, i.e. $\mathbb{N}_q = \{1, 2, \ldots, q\}$. The symbol $\mathbb{I}_n$ denotes the identity matrix of the space $\mathbb{C}^{n \times n}$, while the notation $\mathbb{I}_{n \times m}$ is used to denote a $n \times m$ Hadamard identity matrix (i.e. a $n \times m$ matrix with all its entries equal to 1). The spectrum of a matrix $A \in \mathbb{R}^{n \times n}$, i.e. the set of its eigenvalues, is denoted as $\lambda(A)$. The symbol $\bigoplus$ denotes the direct sum of $n$ matrices, i.e. $\bigoplus_{i=1}^n A_i = \text{diag}(A_1, A_2, \ldots, A_n)$. The notation $\mathbb{R}\{z\}$ and $\mathbb{C}\{z\}$, with $z \in \mathbb{C}$, stands for the real-part and the imaginary-part operators, respectively. The Kronecker product between two matrices $M_1 \in \mathbb{R}^{n \times m}$ and $M_2 \in \mathbb{R}^{p \times q}$ is denoted by $M_1 \otimes M_2 \in \mathbb{R}^{np \times mq}$, while the Kronecker sum of two matrices $P_1$ and $P_2$, with $P_1 \in \mathbb{R}^{n \times n}$ is denoted as $P_1 \oplus P_2$.

### A. WEC Dynamics

We begin this section by recalling from, for example, [15], some well-known basics behind control-oriented WEC modelling, where we assume a 1-degree-of-freedom (DoF) device, to simplify the notation used throughout this paper. In particular, linear potential flow theory is adopted [15], which directly stems from the Navier-Stokes equations, under linear wave theory, and assuming inviscid and incompressible flows.

#### Remark 3: The excitability of (S, L) and the excitation of (S, L).

**Definition 1:** The matrix $Y$ is the moment of system (1) at the signal generator (2).

**Remark 4:** Note that the modelling assumptions considered herein are consistent (and predominant) across a wide variety of WEC control and estimation applications presented in the literature (see [4]).

$^2$We refer the reader to [14] for further detail on the concept of excitability for a general class of systems.

$^3$We note that a similar analysis can be carried out for multi-DoF devices following the moment-based multiple-input, multiple-output framework presented in [9].
The linearised equation of motion of a 1-DoF WEC can be expressed, in the time-domain, as:

$$m \ddot{x} = \mathcal{F}_r + \mathcal{F}_h + \mathcal{F}_c - u,$$

with $m$ the mass of the buoy, $x : \mathbb{R}^+ \rightarrow \mathbb{R}$ the device excursion (displacement), $\mathcal{F}_r : \mathbb{R}^+ \rightarrow \mathbb{R}$ the wave excitation force (external input), $\mathcal{F}_h$ the hydrostatic restoring force, $\mathcal{F}_c$ the radiation force, and $u : \mathbb{R}^+ \rightarrow \mathbb{R}$ the exerted power-take-off (PTO) control force. The hydrostatic force can be written as $\mathcal{F}_h(t) = -s_h \dot{x}(t)$, where $s_h = \rho g D$ denotes the hydrostatic stiffness, with $\rho$ the water density, $D$ the characteristic area of the device, and $g$, the gravitational constant. The radiation force $\mathcal{F}_r$ is modelled using the well-known Cummins’ equation [16], can be written as

$$\mathcal{F}_r(t) = -\mu_x \dot{x}(t) - \int_{\mathbb{R}^+} k(\tau) \dot{x}(t - \tau) d\tau,$$

where $\mu_x = \lim_{\omega \rightarrow +\infty} \tilde{A}(\omega)$, $\mu_x > 0$ represents the added-mass at infinite frequency, $\tilde{A}(\omega)$ is the radiation added mass [15], and $k : \mathbb{R}^+ \rightarrow \mathbb{R}$ is the (causal) radiation impulse response function, containing the memory effect of the fluid response. Finally, the equation of motion of the WEC is

$$\mathcal{M} \ddot{x} + k \dot{x} + s_h x = \mathcal{F}_c - u,$$

with $\mathcal{M} = m + \mu_x$.

Remark 5: The hydrodynamic characteristics of the WEC device under analysis (including the impulse response mapping $k$, $\tilde{A}(\omega)$, and $\mu_x$) can readily be computed using boundary element method solvers, such as NEMOH (open source) [17].

B. Energy-Maximising Control as a Finite-Horizon OCP

To appropriately present our moment-based strategy, we first provide a formal definition of the energy-maximising control problem within a receding-horizon framework.

Let the set of state and input (inequality) constraints be:

$$|x(t)| \leq X_{\text{max}}, \quad |\dot{x}(t)| \leq V_{\text{max}}, \quad |u(t)| \leq U_{\text{max}},$$

with $t \in \mathbb{R}^+$, and where $\{X_{\text{max}}, V_{\text{max}}, U_{\text{max}}\} \subset \mathbb{R}^+$ define the displacement, velocity, and control input limits, respectively. The control objective is to maximise the useful energy absorbed from incoming waves, converted in the PTO system, over a finite-time interval, while consistently respecting physical limits associated with device and actuator dynamics (PTO). This can be written in terms of the following receding-horizon optimal control problem as

$$u_{\text{opt}}^T = \arg \max_{u_T} \frac{1}{T_h} \int_{\Xi_N} u_T(\tau) \dot{x}(\tau) d\tau,$$

subject to:

$$\begin{aligned}
\text{WEC dynamics (5),} \\
\text{state and input constraints (6),} \\
x(t_N^T) = x^m(t_N^T), \\
\dot{x}(t_N^T) = \dot{x}^m(t_N^T),
\end{aligned}$$

with $T_h \in \mathbb{R}^+$ the time-horizon, where we optimise energy-capture within the time-window $\Xi_N = [N \Delta_h, N \Delta_h + T_h] \subset \mathbb{R}^+$, $N > 0$ integer, by means of the control input $u_T{}^T : \Xi_N \rightarrow \mathbb{R}$, and where $\Delta_h$ is denoted as the receding time-step.

Remark 6: Note that the definition of the time-window $\Xi_N$ is strongly linked to the estimation and forecasting requirements of the wave excitation force (see [18]), and the representation of $\mathcal{F}_c$ in moment-domain (see Section IV).

Following Remark 6, we formally write the set $\Xi_N$ as

$$\Xi_N = [N \Delta_h, t_N^m] \cup t_N^m \cup (t_N^N, N \Delta_h + T_h],$$

where $\Xi_N^T$ and $\Xi_N^f$ correspond with past (estimated) and future (forecasted) values of $\mathcal{F}_c$, respectively. The variable $t_N^m \in \Xi_N$ corresponds to the current time instant, which (without any loss of generality) is located in the centre of the time-window $\Xi_N$, i.e. $t_N^m = T_h / 2 + N \Delta_h$. The distribution of the sets and time constants described above is illustrated in Fig. 1. The additional set of (two) equality constraints in (7) are used to guarantee continuity of the state variables $x$ and $\dot{x}$, under the optimal control action $u_{\text{opt}}^T$, where $x^m$ and $\dot{x}^m$ denote the measured values of displacement and velocity, respectively.

We can summarise the receding-horizon OCP of equation (7) in three basic steps:\(^5\)

1) $u_{\text{opt}}^T \leftarrow$ Solve (7) for the time-window $\Xi_N$.
2) Apply $u_{\text{opt}}^m$ in the interval $\Xi_N^T = [N \Delta_h, (N + 1) \Delta_h]$.
3) Replace $\Xi_N$ by $\Xi_{N+1}$ accordingly and go back to 1).

IV. RECEDING-HORIZON MOMENT-BASED APPROACH

Based on the receding-horizon OCP posed in (7), and the theoretical framework developed in [8], [9], we now formally propose a moment-based receding-horizon controller. In particular, Section IV-A discusses the representation of the input $\mathcal{F}_c$ in the moment-domain, for this receding-horizon approach,\(^6\)

\(^4\)Note that equation (5) describes the motion of a generic floating body, under linear potential flow theory.

\(^5\)Note that these steps are standard for any receding-horizon technique [4]
while Section IV-B effectively proposes an energy-maximising real-time controller for WECs.

A. Input Representation in the Moment-Domain

Though highly computationally efficient, a standing assumption for the moment-based control strategy presented in [8], [9] is that the wave excitation input $\mathcal{F}_c$ can be characterised by a periodic mapping. The following framework aims to alleviate the effect of this assumption.

Remark 7: Note that, if the time-horizon $T_h$ is considered to be sufficiently large, then the signal $\mathcal{F}_c$ can be effectively considered $T_h$-periodic, for any practical purposes.

Remark 7 itself poses a contradiction: while the moment-based controller [8], [9] would require a sufficiently large time $T_h$, state-of-the-art forecasting algorithms are not usually able to provide an accurate prediction of $\mathcal{F}_c$ for more than a couple of seconds [12]. In addition, a large $T_h$ can potentially challenge the real-time capabilities of [8], [9].

Motivated by the fact that this periodicity (or sufficiently large $T_h$) condition is limiting in terms of real-time implementation, we introduce, in this section, a methodology to alleviate the effects of this standing assumption. Suppose $\tilde{\mathcal{F}}_{c,n} : \Xi_N \to \mathbb{R}$ denotes the approximated wave excitation input for the time-window $\Xi_N$, composed of both estimated and forecasted values (see Fig. 1). Using the underlying philosophy of the short-term Fourier transform (see [11]), we write the apodised wave excitation force input as

$$[\tilde{\mathcal{F}}_{c,n}]_\vartheta = \vartheta \tilde{\mathcal{F}}_{c,n},$$

where the apodisation mapping $\vartheta : \Xi_N \times \mathbb{R}^+ \to [0,1]$ is used to smoothly bring the excitation force signal, defined for a time-horizon $T_h$, down to zero at the edges of the set $\Xi_N$. This effectively reduces the spectral leakage produced by the discontinuities arising from truncating the signal $\tilde{\mathcal{F}}_{c,n}$ on the (potentially) short time-horizon $T_h$. In other words, the apodised signal $[\tilde{\mathcal{F}}_{c,n}]_\vartheta$ is smoothly brought to zero at the boundaries so that the derivative of its periodic extension is sufficiently smooth. The family of apodisation functions considered herein are the so-called Planck-taper mappings [19]. This set of functions was first suggested within the theory of gravitational waves, and stem from the basic functional form of the Planck distribution, i.e.

$$\vartheta(t, \gamma) = \begin{cases} \frac{1}{1 + e^{-\frac{t-t_N^f}{\gamma}+1}}, & t_N^L \leq t < \gamma t_N^f, \\ 1, & \gamma t_N^f \leq t < (1 - \gamma) t_N^f, \\ \frac{1}{1 - e^{-\frac{t-t_N^f}{\gamma}+1}}, & (1 - \gamma) t_N^f \leq t \leq t_N^L, \\ 0, & t < t_N^L \lor t > t_N^f. \end{cases}$$

where $\{t_N^L, t_N^f\} \subset \Xi_N$ are defined as $t_N^L = N \Delta h + T_h$, and $t_N^f = N \Delta h + T_h$, and the mapping $\mathcal{Z}$ is such that

$$\mathcal{Z}_\pm(t, \gamma) = \begin{cases} \frac{2 \gamma}{1 + (\frac{2 \gamma t}{t_N^L} - 1)}, & t < \gamma t_N^L, \\ \frac{2 \gamma}{1 - 2 \gamma + (\frac{2 \gamma t}{t_N^L} - 1)}, & t > \gamma t_N^L. \end{cases}$$

An example, showing both $\tilde{\mathcal{F}}_{c}$ and $[\tilde{\mathcal{F}}_{c,n}]_\vartheta$ for a time-window $\Xi_N$ and parameter $\gamma = 0.5$, is shown in Fig. 2.

Remark 8: The selection of the set of apodisation functions (10) is motivated by its intrinsic optimality with respect to the preservation of the power spectrum of the wave excitation input (see [19]), being the latter a key variable in the OCP (7). Nevertheless, different apodisation functions can be considered straightforwardly, without further modifications.

Let $\omega_0 \in \mathbb{R}^+$ be the fundamental frequency for the time-horizon $T_h$, i.e. $\omega_0 = 2 \pi / T_h$. Following the theoretical framework proposed in [8], [9], both the apodised wave excitation force $[\tilde{\mathcal{F}}_{c,n}]_\vartheta$ and control input $u$ are expressed in terms of a signal generator described by the set of equations

$$\dot{\xi} = S \xi, \quad [\tilde{\mathcal{F}}_{c,n}]_\vartheta = L_{\xi N} \xi, \quad u = L_{u N} \xi,$$

where $\xi(t) \in \mathbb{R}^\nu$, $\{L_{\xi N}, L_{u N}\} \subset \mathbb{R}^\nu$ and the dynamic matrix $S \in \mathbb{R}^{\nu \times \nu}$ can be written in block-diagonal form as

$$S = \bigoplus_{p=1}^P \left[ \begin{array}{cc} 0 & -p \omega_0 \\ p \omega_0 & 0 \end{array} \right],$$

while $\nu = 2f$, $f > 0$ integer. The initial condition is set to $\xi(0) = \epsilon_\xi$ and the pair $(S, L_{\xi N} - L_{u N})$ is assumed observable.

Remark 9: $\lambda(S) = \{\pm j p \omega_0\}_{p=1}^P$ in (12), i.e. the apodised excitation force is assumed to be a $T_h$-periodic mapping, composed of a finite number $f$ of harmonics of the fundamental frequency $\omega_0$. In particular, the maximum frequency used to describe the apodised excitation force, i.e. the so-called cut-off frequency $\omega_c$, is simply given as $\omega_c = f \omega_0$.

The excitation force input vector $L_{\xi N}$, for a particular time-window $\Xi_N$, can be straightforwardly obtained using a least-squares approach: Let $\mathcal{T}_\xi = \{t_1, \ldots, t_P\} \subset \Xi_N$ be a finite set of $P > \nu$ (integer) uniformly-spaced time instants, and let $\Delta T_\xi \in \mathbb{R}^{\nu \times P}$ and $T_\xi = \mathbb{R}^P$ be defined as

$$\Delta T_\xi = \begin{bmatrix} \xi(t_1) & \ldots & \xi(t_P) \end{bmatrix},$$

$$T_\xi = \mathbf{[} \tilde{\mathcal{F}}_{c,n} \mathbf{]}_\vartheta (t_1, \ldots, t_P).$$

The reader is referred to [11] for further detail on different apodisation mappings.
Then, we define \( L_{\varepsilon_u} \) using the following expression:

\[
L_{\varepsilon_u} := \Lambda [\frac{J}{\varepsilon_u}] \Lambda^T (\Lambda_T \Lambda^T)^{-1},
\]

(15)

where the invertibility of the matrix \( \Lambda_T \Lambda^T \) is guaranteed by the excitability of the pair \((S, \varepsilon_u)\), see [14].

Remark 10: Though real-time performance is already available with (15) (see Section V), we note that, if required, extra computational speed could be achieved using a recursive least-squares implementation instead of (15).

B. Receding-Horizon Moment-Based Controller

Analogously to [8], [9], we can equivalently write the WEC dynamics, expressed in (5), as

\[
\dot{\varphi} = A \varphi - B (k \ast C \varphi) + B (\Phi_e - u), \quad y = C \varphi,
\]

(16)

where \( \varphi(t) = [\varphi_1(t), \varphi_2(t)]^T \) is \( x(t), \dot{x}(t) \in \mathbb{R}^2 \), is the state-vector of the continuous-time model and \( y = \dot{x} \) is the output of the system (assuming velocity to be the measurable device output). Using this representation, the triple of matrices \((A, B, C)\) parameterising equation (16), is given by

\[
A = \begin{bmatrix} 0 & 1 \\ -s_h \mathcal{M}^{-1} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \mathcal{M}^{-1} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}.
\]

(17)

Based on the moment-domain representation of the (apodised) wave excitation force (see Section IV-A), and the equation of motion (16), we now propose a receding-horizon moment-based energy-maximising controller subject to state and input constraints. In particular, under this moment-based framework (and following [8], [9]), the steady-state output response of system (16) can be computed as (see Lemma 1)

\[
y = Y \xi, \quad Y = (L_{\varepsilon_u} - L_{\varepsilon_u}) \Phi^\mathbb{R},
\]

(18)

where \( Y \) is the moment-domain equivalent of the velocity of the device (i.e. the output \( y \) in (16)), and where the matrix \( \Phi^\mathbb{R} \in \mathbb{R}^{\nu \times \nu} \), depending on the system dynamics defined in (16), is defined as

\[
\Phi^\mathbb{R} = (I_\nu \otimes C) \Phi^\mathbb{C}(I_\nu \otimes -B),
\]

\[
\Phi^\mathbb{C} = (S \oplus A) + \mathcal{R} \otimes -BC.
\]

(19)

Finally, the matrix \( \mathcal{R} \), representing the moment-domain equivalent of the radiation force term in (16), is given by

\[
\mathcal{R} = \bigoplus_{j=1}^{\nu} \begin{bmatrix} \Re \{ K(j \omega_0) \} & \Im \{ K(j \omega_0) \} \\ -\Im \{ K(j \omega_0) \} & \Re \{ K(j \omega_0) \} \end{bmatrix},
\]

(20)

with \( K : \mathbb{C}^0 \to \mathbb{C} \) the frequency-response mapping associated with the radiation impulse response function \( k \).

Let \( T_{\nu} = \{ t_i \}_{i=1}^{N_\nu} \subset \Xi_N \) be a set of uniformly-distributed time-instants \(^7\) (collocation points), and define \( \Lambda \in \mathbb{R}^{\nu \times N_\nu} \) and \( \Delta \in \mathbb{R}^{\nu \times 2N_\nu} \) as

\[
\Lambda = \begin{bmatrix} \xi(t_1) & \ldots & \xi(t_{N_\nu}) \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Lambda & -\Lambda \end{bmatrix}.
\]

(21)

\(^7\)See [8], [9] for a discussion on the selection of the set \( T_{\nu} \).

With the definition of \( \Lambda \) and \( \Delta \) in (21), and introducing the moment-domain equivalent of the velocity (18) in the objective function (7), we can write the moment-based control input \( u_{\varepsilon_u}^{\text{opt}} \), for a given time window \( \Xi_N \), as \( u_{\varepsilon_u}^{\text{opt}} = L_{\varepsilon_u}^{\text{opt}} \xi \), where the vector \( L_{\varepsilon_u}^{\text{opt}} \) is the unique global maximiser of the concave QP (see [8], [9]):

\[
L_{\varepsilon_u}^{\text{opt}} = \arg \max_{L_{\varepsilon_u} \in \mathbb{R}^\nu} -\frac{1}{2} L_{\varepsilon_u} \Phi^\mathbb{R}^\mathbb{T} L_{\varepsilon_u} + \frac{1}{2} L_{\varepsilon_u} \Phi^\mathbb{C}^\mathbb{T} L_{\varepsilon_u}
\]

subject to:

\[
L_{\varepsilon_u} A_x \leq B_x, \quad L_{\varepsilon_u} A_y \leq B_y, \quad L_{\varepsilon_u} A_u \leq B_u,
\]

(22)

where the pairs of matrices \((A_x, B_x), (A_y, B_y)\) and \((A_u, B_u)\) are associated with the state and input inequality constraints in (6) on displacement, velocity and control (PTO) input, respectively. In contrast to [8], [9], we now include the pairs of matrices \((A_x^{\text{eq}}, B_x^{\text{eq}})\) and \((A_y^{\text{eq}}, B_y^{\text{eq}})\) to fulfill the equality constraints in (7) at each current time instant, i.e. at \( t = t_i^{\text{eq}} \in \Xi_N \). In particular, the explicit expressions for the pairs of matrices involved in (22) are given by

\[
A_x = -\Phi^\mathbb{R}^\mathbb{T} S^{-1} \Delta, \quad B_x = X_{\max} 1_{\nu \times \nu} + L_{\varepsilon_u} A_x,
\]

\[
A_y = -\Phi^\mathbb{R}^\mathbb{T} S^{-1} \xi(t_i^{\text{eq}}), \quad B_y = x_{\max} 1_{\nu \times \nu} + L_{\varepsilon_u} A_y,
\]

\[
A_u = \Delta, \quad B_u = U_{\max} 1_{\nu \times \nu},
\]

(23)

Remark 11: Following the receding-horizon approach to WEC control discussed in Section III-B, the moment-based optimal control problem, proposed in (22), is solved for a particular time window \( \Xi_N \), and then applied to the system for the time interval \( \Xi_N = [N \Delta h, (N + 1) \Delta h) \), i.e. for a single receding time-step \( \Delta h \). The time window is then subsequently shifted, i.e. \( \Xi_N \mapsto \Xi_{N+1} \), and the process is repeated.

V. CASE STUDY

To demonstrate the performance of the receding-horizon moment-based controller proposed in Section IV, we consider a full-scale state-of-the-art CorPower-like wave energy device, oscillating in heave (translational motion). Fig. 3 presents a schematic illustration of the CorPower-like WEC, along with its corresponding hydrodynamic characterisation,\(^8\) i.e. the frequency-response \( K(j \omega) \) associated with the impulse response mapping \( k \). The dimensions of this device are based on the experimental study performed in [13].

In the remainder of this section, we consider waves generated stochastically from a JONSWAP spectrum [20], with a fixed

\(^8\)\( K(j \omega) \) has been computed using NEMOH (see Remark 5).
significant wave height $H_s$ of 2 [m], peak period $T_p \in [5, 12]$ [s] and peak shape parameter $\gamma = 3.3$. Since the waves are generated from sets of random amplitudes [21], it is found that a mean of $\approx 40$ simulations (per sea state) is necessary to obtain statistically consistent performance results for the controller presented in this study.

Remark 12 (On the selection of $T_h$): The time-horizon $T_h$, which characterises the time-window $\Xi_N$, has to be selected bearing in mind the following trade-off. While a larger $T_h$ (i.e. a smaller fundamental frequency $\omega_0$) implies a more refined description of the apodised excitation input, with respect to the least-squares procedure described in (15) (see Remark 9), an increased number of harmonics $f$ is required to reach a given cut-off frequency $\omega_c$, which naturally increases the computational requirements of the strategy (note that the order of the matrices involved in the QP problem (22) directly depends on $f$). This trade-off can be simply assessed by means of numerical analysis.

For this case study, and following Remark 12, the time-horizon is selected as $T_h = 60$ [s], i.e. we consider 30 [s] of both estimated and forecasted values of $F_e$. This corresponds to a fundamental frequency $\omega_0 = 2\pi/60$ [rad/s], which provides accurate results with respect to the least-square procedure described in (15), with mild computational requirements. The receding time-step is fixed as $\Delta_h = 0.1$ [s], while the dimension (order) of the signal generator (12) is chosen as $\nu = 60$ (corresponding with a cut-off frequency of $\omega_c \approx 6$ [rad/s]). With respect to state constraints, we fix the maximum allowed displacement and velocity values as $X_{\text{max}} = 2$ [m] and $V_{\text{max}} = 2$ [m/s], respectively.

Remark 13: We note that the moment-based controller runtime, i.e. the time required to compute the energy-maximising optimal control input for the duration of the receding-step $\Delta_h$, is in the order of $\sim 1$ [ms] $\ll \Delta_h$ for the totality of the preceding simulations (implemented in MATLAB), hence always easily achieving real-time performance. Naturally, the speed at which computations are performed can be further improved (if required) by simply implementing this algorithm in a compiled language, such as C or C++.

As discussed in Section I, the unknown-input estimation strategy, selected to compute the estimation section of $\bar{F}_e\mathcal{N}$, for each time-window $\Xi_N^r$, is based on a combination of Kalman filtering and the internal model principle of control theory, as presented (and tuned) in [12]. The forecasting algorithm considered, over the set $\Xi_N^f$, is the AR model proposed in [3], where the order $\mathcal{O}$ is set to 200 (see Section V-C).

A. Results and Discussion

Initial controller performance assessment focusses on energy absorption under both displacement and velocity constraints. Fig. 4 shows absorbed energy for sea states with $H_s = 2$ [m] and $T_p \in [5, 12]$, where the displacement and velocity of the CorPower-like device are constrained to $X_{\text{max}}$ and $V_{\text{max}}$, respectively. The solid-black line represents the ideal performance scenario, where the excitation force is assumed to be perfectly known over each time-window, i.e. $F_{e\mathcal{N}} = F_{e\mathcal{N}}$, $\forall N$, while the dotted-grey line shows the actual performance of the moment-based controller, where we utilise the approximated excitation force $\bar{F}_e\mathcal{N}$, computed with both estimation and forecasting algorithms. Clearly, the actual performance of the proposed receding-horizon moment-based approach is almost indistinguishable from its ideal counterpart, being able to perform optimally, with differences of less than 5% in terms of energy absorption. Fig. 4 also includes absorbed energy obtained with a reactive (displacement and velocity) feedback, used here as benchmark strategy (see [15]). The feedback gain is computed using exhaustive search, ensuring that the specified constraints are met for each sea-state analysed (as in, for instance, [22]). Note that the receding-horizon moment-based strategy is consistently outperforming the feedback controller in terms of energy-absorption, for the totality of the sea states.

To fully illustrate the capabilities of the proposed strategy, Fig. 5 presents time traces of displacement (a, left axis, solid-black), velocity (a, left axis, dashed-black) and control input (b, left axis, solid-black), for a specific sea-state realisation with $T_p = 8$ [s], where we also include a maximum control (PTO) force constraint $U_{\text{max}} = 1 \times 10^6$ [N]. We note that some key features can be directly appreciated from Fig. 5, which we discuss in the following. To begin with, the state and input limits, under the action of the receding-horizon moment-based control strategy, are being consistently respected $\forall t$, hence illustrating the capability of the approach to maximise energy absorption for
WECs, while respecting the physical limitations of both device and actuator (PTO). Moreover, we note that, even in this fully constrained case, the velocity of the device under optimal control conditions remains 'in-phase' with the estimated wave excitation force, over the time-interval $\Xi_N$. From now on, aiming to simplify the presentation of results, we fix the wave peak period to $T_p = 8$ s, given that almost identical conclusions can be drawn using different values for $T_p \in [5,12]$.

As discussed in the comparison study [2], there are two main sources of errors affecting $\tilde{F}_{eN}$, arising from improper tuning of any unknown-input estimator: Constant errors in instantaneous amplitude (i.e. constant deviations in envelope), and instantaneous phase (i.e. time-delays). We represent these imperfections as error sources affecting the estimation stage using the criterion specified in what follows.

Remark 14: Another possible error source is the presence of measurement noise, i.e. the estimator is tuned in such a way that high frequency noise (affecting motion sensors) is not filtered. This effect is not analysed herein: Note that the moment-based representation for the input (discussed in Section IV-A) can intrinsically filter high frequency components, via a suitable selection of $\nu$ in (13).

Let $\{F_\alpha, F_\phi\} \subset \Omega_F$, with $\Omega_F = [0.75, 1.25]$, be (error) factors associated with amplitude (A) and phase (P) of $\tilde{F}_{eN}$. We analyse, for $t \in \Xi_N$, the following error sources:

\[
\begin{align*}
A & : \tilde{F}_{eN}(t) \rightarrow F_\alpha \tilde{F}_{eN}(t), \\
\text{P} & : \tilde{F}_{eN}(t) \rightarrow F_\phi \tilde{F}_{eN}(t + (F_\phi - 1)T_p), \\
\text{A+P} & : \tilde{F}_{eN}(t) \rightarrow F_\alpha F_\phi \tilde{F}_{eN}(t + (F_\phi - 1)T_p).
\end{align*}
\]

(24)

Case A assumes that the amplitude of the estimated signal is not estimated correctly, i.e. $\tilde{F}_e$ is multiplied by a factor $F_\alpha$, while case P effectively considers the existence of a phase (delay) between estimated and true excitation force, proportional to the peak period $T_p$. Lastly, case A+P combines both sources of error, by assuming that the estimated excitation force has both amplitude and phase errors, for all possible combinations of $(F_\alpha, F_\phi)$ in $[0.75,1.25]$.

Remark 15: Due to the underlying linearity of the AR model considered in this study, if $\tilde{F}_{eN}$ is modified either by scaling, shifting in time, or superposing both cases, for $t \in \Xi_N$, this modification propagates within the forecasted time-window $\Xi_f$ in the exact same manner. In other words, the sources of estimation error described in cases A, P and A+P affect the forecasted signal in the exact same proportions.

Fig. 6 presents an illustrative example of an excitation force signal affected by cases A and P, for a time-window $\Xi_N$. In particular, we show the estimated and forecasted excitation force with $F_\alpha = F_\phi = 1$, i.e. error-free (solid-black), and for various values of $F_\alpha$ (a, green) and $F_\phi$ (b, green). Taking into account the cases defined in (24), the sensitivity analysis with respect to estimation errors is defined in terms of a suitable performance indicator $R_{J_{\tilde{F}_E}}$. In particular, we define $R_{J_{\tilde{F}_E}}$ as $R_{J_{\tilde{F}_E}}(F_\alpha, F_\phi) = J'(F_\alpha, F_\phi)/J'(1,1)$, where the image of the mapping $J: \Omega_F \times \Omega_F \rightarrow \mathbb{R}$ is the absorbed energy throughout the complete simulation time, for any pair of values $(F_\alpha, F_\phi)$, under controlled conditions.

Remark 16: To be precise, $R_{J_{\tilde{F}_E}}$ is the ratio between the absorbed energy under the moment-based control strategy (22), with and without the presence of estimation errors. In other words, the performance indicator $R_{J_{\tilde{F}_E}}$ describes how optimal energy absorption is affected if the unknown-input estimator is
Fig. 7. $R_f^c$, for cases A (a) and P (b). A value of $R_f^c$ below zero (solid-blue line) indicates negative energy absorption. The filled black circle indicates absorption with $F_\alpha = F_\phi = 1$.

Fig. 8. $R_f^c$, for case A+P. A value of $R_f^c$ below zero (solid-blue line) indicates negative energy absorption. Cases A and B are depicted with solid-black (empty circle) lines. The filled black circle indicates absorption with $F_\alpha = F_\phi = 1$.

not perfectly tuned, distinguishing explicitly between amplitude and phase errors.

Fig. 7 shows performance results for cases A (a) and P (b), in terms of $R_f^c(F_\alpha, 1)$ and $R_f^c(1, F_\phi)$, respectively. For case A, it can be appreciated that, even under an amplitude deviation of ±25% from its true value, the absorbed energy always remains above 90% of its optimal achievable performance (computed without any estimation amplitude or phase errors). In other words, deviations in amplitude, for the estimated wave excitation force, generate only small deviations in absorbed energy under controlled conditions. This is clearly not the case for phase deviations, i.e. case P, where a delay (positive or negative) of ≈10% of the peak period (around 0.8 [s], for this case study), not only dramatically affects optimal energy absorption, but actually generates negative power (the device starts to drain energy from the electric grid). This clearly indicates that maximal effort should be put into tuning the estimator to guarantee phase synchronisation with the target wave excitation signal, hence achieving optimal energy-maximisation, under controlled conditions.

Finally, Fig. 8 shows results for case A+P, where both errors in amplitude and phase are analysed simultaneously. Similarly to case P, it is clear that the presence of a time-delay (positive or negative) has a much greater impact on energy absorption than any existing differences in estimated amplitude. Interestingly, while positive or negative delays have an almost symmetric effect, underprediction of the wave excitation force amplitude has a lesser impact on performance than overprediction. Note that this behaviour is consistent with that of Fig. 7(a).

C. Sensitivity Analysis: Forecasting

We now consider errors arising purely from the forecasting procedure, i.e. we assume that the unknown-input estimator is well-tuned (achieving convergence towards the target excitation force), and that any potential mismatch is only present within the forecasted window $\Xi_N$. To that end, we define the performance indicator $R_f^t(t^f) = J(t^f)/J(5)$, where the image of the operator $J : R^+ \to R$ is the energy absorbed, assuming $t^f < 5$ seconds of forecast within 99% and 100% of accuracy.

Remark 17: $R_f^t$ is the ratio of absorbed energy, under controlled conditions, between energy extraction assuming quasi-perfect knowledge of the forecasted signal for a section of $\Xi_N$, and the maximum time-length with achievable forecast with more than 99% of accuracy, i.e. ≈ 5 [s], obtained from a sufficiently large AR model order $O$ (here chosen as $O = 200$). In other words, $R_f^t$ describes how optimal energy absorption is affected in terms of the accuracy of the forecasting algorithm.

Fig. 9 presents performance results in terms of the indicator $R_f^t(t^f)$, while Fig. 10 presents an illustrative example of a forecasted excitation force signal with $t^f = 5$ [s] (dashed-black), and for $t^f < 5$ [s] (dashed-green). The target excitation force $F_e$ is depicted with a solid-grey line.

Authorized licensed use limited to: Maynooth University Library. Downloaded on December 18,2020 at 10:33:37 UTC from IEEE Xplore. Restrictions apply.
VI. CONCLUSION

This paper proposes a real-time receding-horizon energy-maximising controller based on moments, by formally extending [8], [9] to a finite-horizon OCP. The representation of the wave excitation force input, in the moment-domain, is adapted to short prediction horizons by the use of a convenient family of apodisation mappings. The optimal control objective is modified accordingly, using a receding-horizon approach, while retaining the intrinsic computational efficiency and uniqueness of the energy-maximising solution provided by the moment-based framework. We show that the strategy proposed in this paper, which incorporates both wave excitation force estimation and forecasting, is able to perform to virtually ideal levels, where full-knowledge of $F_{w}$ is available over the time-horizon, with differences of less than 5% in terms of power absorption for the range of analysed sea-states. In addition, we present a sensitivity analysis addressing the impact of estimation and forecasting errors on total energy absorption under controlled conditions. Two main conclusions can be directly extracted from this analysis: Forecasting mismatches have a negligible impact on the overall performance of the strategy, while differences arising from unknown-input estimators can effectively generate negative power absorption. In particular, phase errors (positive or negative) in the estimated excitation force have a substantial impact on the energy-maximising performance of the controller, suggesting that maximal design effort should be put in tuning the observer such that (instantaneous) phase synchronisation is achieved with the target excitation force.

REFERENCES


