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Homogeneous Filtering Unknown Input Observer for Wave Energy Applications

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***Abstract*— In wave energy systems, in contrast to other renewable energy applications, the external input acting on the system is not a measurable quantity. Simultaneously, knowledge of this wave excitation force is essential for the operation of energy-maximising controllers. Considering these aspects, the problem addressed in this paper is that of the design of an unknown input observer, based on homogeneous sliding mode techniques, which is capable of simultaneously rejecting unbounded measurement noise and providing a robust estimate of the unknown input to wave energy systems, the excitation force. While inspired by a wave energy application, the proposal is generic, requires only a measurement of the system output, and provides precise estimation of the system states and the unknown input. The precision of the unknown input estimation is conditioned by the accuracy of the employed model, but, notably, the estimation errors associated with the homogeneous part of the observer are asymptotically optimal, guaranteeing exact convergence in the absence of measurement noise.**

I. Introduction

Wave energy conversion (WEC) systems are oscillating structures capable of extracting energy from the motion of waves and it has been shown that advanced control techniques can significantly improve their commercial viability [1]. WECs oscillate as the result of fluid-structure interactions, and major hydrodynamic differences exist between different WEC prototypes. Analytically, a non-parametric model for the WEC may be obtained by solving a surface integral over the wetted surface of the WEC [2]. However, for the design of energy-maximising controllers, a parametric model is required. In this context, the most widespread linear control-oriented model can be represented as:

$$m\ddot{x} = \underbrace{f_e - f_u}_{\text{External forces}} \underbrace{- f_r(\dot{x}, \ddot{x}) - f_k(x)}_{\text{Internal forces}}, \tag{1}$$

where x is vertical displacement (assuming a one-degree-of-freedom device), m is the WEC mass, f_r and f_k are the radiation and hydrostatic restoring forces that define the free system dynamics, and f_e and f_u are the two external forces acting on the body. Here, f_e is the wave excitation force, and f_u is the control action, which is calculated to maximise energy extraction from f_e. Energy-maximising controllers essentially require obtaining the panchromatic complex-conjugate of the WEC intrinsic impedance [2], which is defined in the Fourier domain from $f_r(\dot{x}, \ddot{x})$ and $f_k(x)$. In this scenario, optimal WEC controllers require a forecast of the excitation force to define an energy-maximising trajectory [3]. However, due to the complex nature of fluid-structure interactions, f_e is not easily separable from other hydrodynamic forces on the WEC device. Consequently, f_e is an unmeasurable quantity, and real-time estimation is necessary.

In the literature, there have been multiple proposals to estimate the excitation force [4], [5], [6]. Notably, Kalman filter (KF) based algorithms are the most widespread when using only WEC state measurements, e.g., position and velocity [4]. This is mainly because of their ability to deal with measurement noise and model uncertainty [7]. A relatively unexplored alternative is based on filtering sliding-mode (SM) differentiators (FSMD) [8]. Therefore, in this paper, inspired by the excitation force estimation problem, a new unknown-input observer structure, based on FSMDs, is proposed. The proposed estimator, termed a homogeneous SM-based observer (HSMO), consists of an SM filter capable of rejecting unbounded noise, provided this is small on average (formalised in Section II), and a homogeneous SM structure designed to mimic the system dynamics. The results show that this structure is capable of matching the results obtained with a KF-based unknown input observer but with additional design simplicity and improved convergence dynamics, due to the finite-time convergence of SM algorithms.

A. Notation

Throughout this paper, vectors and matrices are indicated with bold characters, such as $\mathbf{u}$, $\boldsymbol{x}$, or $\mathbf{A}$, while scalar elements are indicated with lower-case italic characters, such as k, l, m. $\mathbb{R}^{n\times m}$ denotes real matrices with n rows and m columns. Additionally, $\oplus$ indicates the direct sum of matrices, dynamical systems are concisely written with Σ_α, $||\boldsymbol{x}||_\infty$ stands for the infinite norm of $\boldsymbol{x}$, and $\lfloor\cdot\rceil^\alpha := |\cdot|^\alpha \text{sign}(\cdot)$. $f^{(k)}(t)$ represents the k-th time derivarive of $f(t)$. Also, the time argument is omitted when clear from the context.

B. Paper organisation

The present work is organised as follows. In Section II the required preliminary definitions are presented. In Section II-B a linear WEC model, employed for the design of the HSMO, is presented. The main contribution, the HSMO, is developed in Section III, and a numerical example focused on the f_e estimation is developed in Section IV. Finally, in Section V, the main conclusions are drawn.

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II. PRELIMINARIES

In this section, the required definitions and the WEC model used throughout the paper are defined.

A. Fundamental definitions

First, a differential inclusion (DI) is denoted as:

$$\dot{\boldsymbol{x}} \in \boldsymbol{\Phi}(\boldsymbol{x}), \quad \boldsymbol{\Phi}(\boldsymbol{x}) \subset T_x\mathbb{R}^{n_x}, \tag{2}$$

where its solutions are locally absolutely continuous functions $\boldsymbol{x}$, that satisfy (2) for almost all t. $T_x\mathbb{R}^{n_x}$, denotes the tangent space to $\mathbb{R}^{n_x}$ for $\boldsymbol{x} \in \mathbb{R}^{n_x}$. Equation (2) is a Filippov DI [9] if the vector field $\boldsymbol{\Phi}(\boldsymbol{x}) \in T_x\mathbb{R}^{n_x}$ is nonempty, upper semicontinuous, and compact and convex for any $\boldsymbol{x}$.

A coordinate dilation in $\mathbb{R}^{n_x}$, with weights $m_1, m_2, \ldots, m_{n_x}$ is defined as:

$$\begin{aligned} d_\kappa : &(x_1, x_2, \ldots, x_{n_x}) \mapsto \\ &(\kappa^{m_1}x_1, \kappa^{m_2}x_2, \ldots, \kappa^{m_{n_x}}x_{n_x}), \end{aligned} \tag{3}$$

with $\kappa \geqslant 0$, which is an expansion or contraction of the coordinate state-space vector $\boldsymbol{x}$. Employing (3), it is possible to define the weighted homogeneity degree for DI (2).

Definition 1: A vector field $\boldsymbol{\Phi}(\boldsymbol{x}) \subset T_x\mathbb{R}^{n_x}$, $\boldsymbol{x} \in \mathbb{R}^{n_x}$, from a DI (2), is homogeneous of degree $q \in \mathbb{R}$ if:

$$\boldsymbol{\Phi}(\boldsymbol{x}) = \kappa^{-q} d_\kappa^{-1}\left(\boldsymbol{\Phi}(d_\kappa \boldsymbol{x})\right), \tag{4}$$

with $\boldsymbol{x} \neq 0$ and $\kappa > 0$, $\kappa \in \mathbb{R}$, holds. That is, the vector field is invariant w.r.t. the transformation $\kappa^{-q} d_\kappa^{-1}\left(\boldsymbol{\Phi}(d_\kappa \boldsymbol{x})\right)$.

Employing dilation (3) in (4) permits any system dynamics with any homogeneity degree $\neq 0$, to be scaled to obtain a homogeneity degree ± 1. Homogeneity has been a stepping stone in recently developed sliding mode-based algorithms, since a negative homogeneity degree guarantees finite time convergence [10]. Furthermore, since Filippov DIs feature the existence and extendability of the results, local convergence implicitly indicates global convergence, removing the necessity of Lyapunov-based stability demonstrations [11]. In this paper, all differential equations are understood in the Filippov sense.

Definition 2: A function $\nu(t)$, $\nu : [0, \infty) \to \mathbb{R}$ is called a signal of *global filtering* order k, $k \geqslant 0$ if ν is a locally integrable Lebesgue measurable function, and there exists a solution ξ for the differential equation $\xi^{(k)} = \nu$. Then, $|\xi(t)|$ is the k-th global order integral magnitude of ν.

Definition 3: Any signal $\nu(t)$, $\nu : [0, \infty) \to \mathbb{R}$ is termed *locally filterable* if it can be represented as $\nu(t) = \varepsilon_0 + \varepsilon_1 + \ldots + \varepsilon_k$, where each ε_i, with $i = 0, 1, \ldots, k$, are signals of global filtering order $0, 1, \ldots, k$, respectively. Note that locally filterable functions only satisfy Definition 2 over finite length intervals. That is, for positive constants t_1, T, there exists a solution $\xi(t)$, $t \in [t_1, t_1 + T]$ for $\xi^{(k)}(t) = \nu(t)$, with local $(k-l)$ filtering order a_l, satisfying $|\xi^{(l)}(t)| < a_l$, with $l = 0, 1, 2, \ldots k-1$.

The global and local filtering order are essential definitions, provided FSMDs are capable of rejecting noise with a finite global (or local) filtering order when their integral magnitude is negligible when compared with the differentiated signal. For a broader discussion about these definitions, please refer to [8], [12], [13].

B. Wave energy conversion system model

A linear WEC model (1), requires definitions for the radiation and restoring forces. Firstly, assuming small displacement from an equilibrium position:

$$f_k = k_x x, \tag{5}$$

where k_x is the hydrostatic restoring force coefficient. The radiation force may be defined in terms of a non-parametric linear convolution [14]:

$$f_r = m_\infty \ddot{x} + \underbrace{\int_{t_0}^{t} h_r(t-\tau)\dot{x}(\tau)d\tau}_{\tilde{f}_r}, \tag{6}$$

where m_∞ is the infinite frequency added-mass, x, $\dot{x} := v$, and $\ddot{x} := \dot{v}$ represent the displacement, velocity and acceleration of the WEC, respectively, and h_r is the radiation impulse response kernel. Because of the nature of the system, the convolution operator describes a causal strictly passive system. Additionally, to approximate the convolution from (6), a linear, continuous-time, strictly proper, finite-dimensional system, is considered:

$$\Sigma_r : \begin{cases} \dot{\mathbf{x}}_r = \mathbf{F}\mathbf{x}_r + \mathbf{G}v, & \text{(7a)} \\ \tilde{f}_r \approx \mathbf{H}\mathbf{x}_r, & \text{(7b)} \end{cases}$$

with $\mathbf{F} \in \mathbb{R}^{n_r \times n_r}$ Hurwitz, and $\mathbf{G} \in \mathbb{R}^{n_r \times 1}$ and $\mathbf{H} \in \mathbb{R}^{1 \times n_r}$. Thus, $\mathbf{x}_r^\intercal = [x_{r_1}, \ldots, x_{n_r}]$. For a formal discussion on the properties associated with Σ_r, see [15]. Considering (5) to (7), the approximate state-space representation of (1) is:

$$\Sigma_w : \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\left(f_e - f_u\right), & \text{(8a)} \\ y = \mathbf{C}\mathbf{x}, & \text{(8b)} \end{cases}$$

where $\mathbf{x}^\intercal = [x, v, \mathbf{x}_r^\intercal]$, $y = x$, and the triple which represents (1), in state space form, is:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_M & -\mathbf{B}_M\mathbf{H} \\ \mathbf{G}\mathbf{C}_M & \mathbf{F} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{B}_M \\ \mathbf{0} \end{bmatrix}, \mathbf{C}^\intercal = \begin{bmatrix} \mathbf{C}_M \\ \mathbf{0} \end{bmatrix}, \tag{9}$$

$$\mathbf{A}_M = \begin{bmatrix} 0 & 1 \\ -\mathcal{M}k_x & 0 \end{bmatrix}, \mathbf{B}_M = \begin{bmatrix} 0 \\ \mathcal{M} \end{bmatrix}, \mathbf{C}_M^\intercal = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{10}$$

where $\mathcal{M} = (m + m_\infty)^{-1}$, and the zero vectors are $\mathbf{0} \in \mathbb{R}^{n_r \times 1}$.

III. UNKNOWN INPUT OBSERVER DESIGN

The main focus of this paper is to design an unknown input observer, capable of recovering not only the system states but also the unknown input, i.e., the wave excitation force (f_e). In this paper, the proposed HSMO is based on a variation of the FSMD [8][13]. The goal is to utilise only measurements of the device position x (alternatively, velocity), assuming that $x = x_0 + \varepsilon$, where x_0 is the base signal to differentiate and ε is a high-frequency noise component.

The FSMD structure is selected due to its finite time convergence and insensitivity to noise as detailed in 1) below

[12], [16], [17], [18]. Specifically, FSMDs are capable of providing up to the n_{th} time derivative of an input signal if:

1) The noise ε contained in the signal to differentiate ε is a signal of global (or local) filtering order $k \geqslant 0$, with k-th order integral magnitude $\epsilon \geqslant 0$. That is, ε may be possibly unbounded but with a small local average value. Note that as a particular case, with $k = 0$, the requirement is $|\varepsilon| \leqslant \epsilon$ with constant ϵ.
2) There exists a known Lipschitz bound for the n-th time derivative of the base signal to differentiate x_0.

Although the proposal presented in [8] is to differentiate signals, here it is proven that an extension of this structure provides estimates not only of the system states but also of the unknown input [19].

A. Homogeneous sliding mode observer (HSMO)

In essence, the proposed HSMO is a mimic of the original system, which is built assuming that a measurement of x is available. The proposed observer possesses an SM-based filter, capable of rejecting unbounded noise of filtering order n_f, and a small integral order magnitude. In addition, it is capable of providing both an estimate of the WEC states and an estimate of f_e. To that end, it is formulated as:

$$\Sigma_o : \begin{cases} \dot{w}_1 = w_2 + \phi_{2+n_f}(w_1), & \text{(11a)} \\ \dot{w}_2 = w_3 + \phi_{2+n_f-1}(w_1) & \text{(11b)} \\ \quad\vdots & \\ \dot{w}_{n_f-1} = w_{nf} + \phi_4(w_1) & \text{(11c)} \\ \dot{w}_{n_f} = \hat{x} - x + \phi_3(w_1), & \text{(11d)} \\ \dot{\hat{x}} = \hat{v} + \phi_2(w_1), & \text{(11e)} \\ \dot{\hat{v}} = \tilde{f}_e - \dfrac{\hat{\tilde{f}}_r + \hat{f}_k + f_u}{m_0} + \phi_1(w_1), & \text{(11f)} \\ \dot{\tilde{f}}_e = \phi_0(w_1), & \text{(11g)} \\ \dot{\hat{\mathbf{x}}}_r = \mathbf{F}\hat{\mathbf{x}}_r + \mathbf{G}\hat{v} & \text{(11h)} \\ \hat{\tilde{f}}_r = \mathbf{H}\hat{\mathbf{x}}_r & \text{(11i)} \\ \hat{f}_k = k_x\hat{x}. & \text{(11j)} \end{cases}$$

where equations (11a) to (11d) constitute a n_f-order SM filter, designed to integrate the error $e_x = \hat{x} - x$, and reject spurious components of the filtering order not exceeding n_f. Each $\phi_i(w_1)$ is defined as $\phi_i(w_1) = \lambda_i L^{(3+n_f-i)/(3+n_f)} \lfloor w_1 \rceil^{i/(3+n_f)}$, with $i = 0, 1, ..., 2 + n_f$, λ_i are predefined fixed gains, and L is a Lipschitz bound-based constant formally defined in Subsection III-B. The constant L is the only adjustable parameter used to guarantee algorithm convergence. Finally, (11e)-(11j) represent a copy of the model Σ_w of (8), assuming that m_0 is the nominal mass of the WEC and that the real mass m may be unknown. To prove convergence of (11), the errors $e_0 = e_x = \hat{x} - x$, $e_1 = e_v = \hat{v} - v$, and

$$e_2 = e_{f_e} = \tilde{f}_e - \frac{f_e}{m} \underbrace{- \frac{\hat{\tilde{f}}_r + \hat{f}_k + f_u}{m_0} + \frac{f_r + f_k + f_u}{m}}_{\Delta_{\Sigma f}}, \quad (12)$$

are defined, and the error dynamics of the first $3 + n_f$ states of the HSMO are rewritten as a Fillipov DI:

$$\mathbf{\Phi}(\boldsymbol{e}) : \begin{cases} \dot{w}_1 = w_2 + \phi_{2+n_f}(w_1), & \text{(13a)} \\ \quad\vdots & \\ \dot{w}_{n_f} = e_x + \phi_3(w_1), & \text{(13b)} \\ \dot{e}_0 = \dot{e}_x = e_v + \phi_2(w_1), & \text{(13c)} \\ \dot{e}_1 = \dot{e}_v = e_{f_e} + \phi_1(w_1), & \text{(13d)} \\ \dot{e}_2 = \dot{e}_{f_e} \in \phi_0(w_1) + [-L;\, L], & \text{(13e)} \end{cases}$$

with $\boldsymbol{e} := [w_1, ..., w_{n_f}, e_0, e_1, e_2]^\intercal$, and assuming $|\frac{d}{dt}\left(\frac{f_e}{m} + \Delta_{\Sigma f}\right)| < L$. Then, it can be appreciated that (i) By applying the dilation:

$$d_\kappa : (w_1, ..., w_{n_f}, e_0, e_1, e_2) \mapsto (\kappa^{3+n_f} w_1, ..., \kappa^4 w_2, \kappa^3 e_0, \kappa^2 e_1, \kappa^1 e_2), \quad (14)$$

and (ii) Considering that $\phi_i(\kappa^{3+nf} w_1) = \kappa^i \phi_i(w_1)$, the DI, $\mathbf{\Phi}(\boldsymbol{e})$, is homogeneous with homogeneity degree $= -1$. This is verified by evaluating (4) employing the dilation (14):

$$\kappa^{-q} d_\kappa^{-1}\left(\mathbf{\Phi}(d_\kappa \boldsymbol{e})\right) = \kappa^{-q}\kappa^{-1}\left(\mathbf{\Phi}(\boldsymbol{e})\right) = \mathbf{\Phi}(\boldsymbol{e}), \quad (15)$$

which is satisfied only if $q = -1$. As a consequence, if there exists L satisfying $|\frac{d}{dt}\left(\frac{f_e}{m} + \Delta_{\Sigma f}\right)| < L$, the convergence of (13) is independent of the (bounded) radiation $\hat{\tilde{f}}_r$ and restoring $\hat{f}_k$ forces estimates, and finite-time convergence of the estimates $\hat{x} \to x$, $\hat{v} \to v$ and $\tilde{f}_e \to \frac{f_e}{m} + \Delta_{\Sigma f}$ is guaranteed. Regarding the dynamics of the radiation states error ($\mathbf{e}_{\mathbf{x}_r}$), these are given by:

$$\dot{\mathbf{e}}_{\mathbf{x}_r} = \mathbf{F}\mathbf{e}_{\mathbf{x}_r} + \mathbf{G}e_v. \quad (16)$$

Therefore, provided $e_v \to 0$ in the absence of measurement noise, $e_{\mathbf{x}_r}$ asymptotically converges to zero. This is because the radiation subsystem is strictly passive [2] and, thus, $\mathbf{F}$ is Hurwitz.

A noticeable aspect, however, is that $\tilde{f}_e$ does not converge to f_e/m whenever there is model uncertainty. This is because the unknown input can only be estimated with knowledge of the system. Therefore, the term $\Delta_{\Sigma f}$ must be considered to include model uncertainty and initial conditions errors.

The HSMO convergence is guaranteed provided that their design coefficients are adequately tuned. Thus, it is also important to consider the effects of the gains λ_i and the Lipschitz-based gain in the estimation errors in the presence of measurement noise as defined below.

Assumption 1: The noise ε contained in the measured signal $x = x_0 + \varepsilon$, is composed of n_f components:

$$\varepsilon = \varepsilon_0 + \varepsilon_1 + \ldots + \varepsilon_{n_f}, \quad (17)$$

and each term ε_i, $i = 1, 2, ..., n_f$ is of global filtering order i with i-th order integral magnitude ϵ_i. Additionally, the observer is assumed to operate in $t \in \mathbb{R}$.

Employing Assumption 1 and [8], the asymptotically optimal accuracy of the HSMO is given by:

$$|e_j| \leqslant \pi 2^{\frac{j}{3}-1} L\rho^{3-j} \leqslant \mu_j L\rho^{3-j}, \text{ with } j = 0, 1, 2, \quad (18)$$

$$|e_{\mathbf{x}_r}| \leqslant |e_1| \; ||\mathbf{G}||_\infty, \tag{19}$$
$$|w_1| \leqslant \mu_{w_1} L \rho^{3+n_f}, \tag{20}$$
$$\rho = \max\left[\left(\frac{\epsilon_0}{L}\right)^{1/3}, \left(\frac{\epsilon_1}{L}\right)^{1/4}, ..., \left(\frac{\epsilon_{n_f}}{L}\right)^{1/(3+n_f)}\right], \tag{21}$$

where μ_j and μ_{w_1} are fixed, bounded from below with the Kolmogorov constants [8], and only depend on the selection of the λ_i gains with $i = 0, 1, ..., 2 + n_f$. Since the selection of the λ_i affects the accuracy asymptotics of the proposed observer, these may be selected following [8], where a recommended set of gains λ_i is presented. On its part, to guarantee the HSMO convergence, L must satisfy the design criteria specified in Subsection III-B. Complementary, in the case of a finite sampling rate, recently, discretisation proposals that do not deteriorate the asymptotic accuracy of the differentiator have been made [20].

B. Lipschitz-based gain selection

As previously mentioned, the convergence of the HSMO only depends on the appropriate selection of a large enough gain, L, based on a Lipchitz bound. More specifically, for the WEC under study, considering (12) and (13e):

$$\left|\frac{d}{dt}\left(\frac{f_e}{m} + \Delta_{\Sigma f}\right)\right| \leqslant \left|\frac{\dot{f}_e}{m}\right| + \left|\dot{\Delta}_{\Sigma f}\right| < L. \tag{22}$$

Therefore, to guarantee the convergence of the proposed HSMO, an L satisfying (22) is required. To find L, several considerations must be made. In the first place, since resource characterisation is a mature area of research, bounds for $\dot{f}_e/m$ (which is the principal contributor to L) are easily accessible [21]. In the second place, the contribution of the term $\dot{\Delta}_{\Sigma f}$ is relatively negligible when a good model is available. Otherwise, a bound for this term may be found theoretically, considering the unmodelled hydrodynamic effects, although it is not a trivial task. Hence, in practice, it is advisable to resort to an in-silico approach, analysing different initial conditions errors and unmodelled dynamics. Moreover, although beyond the scope of this paper, an adaptive Lipchitz gain algorithm may be designed [13].

Finally, it is worth considering that a larger L reduces the initial convergence time of the observer at the expense of increasing the error bounds (as can be observed in (18)-(21)). Nevertheless, in wave energy applications, the convergence time is not a critical parameter due to the slow dynamics of wave climate variations [21]. Thus, the smallest L satisfying (22) can be selected providing the lowest error bounds while guaranteeing convergence.

IV. Results and Discussion

In this section, numerical results, obtained using the HSMO in a WEC application, are presented. To emulate a realistic environment, the device considered for the observer evaluation is a heaving point absorber buoy [22], with parameters as presented in Table I. This is a one-degree-of-freedom device, which satisfies (8). For the design of the HSMO, it is assumed that the measurement noise is of filtering order < 2, thus, $n_f = 2$. Additionally:

- Two measurements are available, position and velocity. Thus, two separate HSMOs are independently designed; one to filter and estimate position, and the other to estimate velocity and reconstruct f_e.
- The HSMOs are discretised employing the proposal from [13], with sampling period $T = 1 \cdot 10^{-4}$.

As a result, the HSMO for the velocity and f_e reconstruction is given by:

$$\Sigma_{\hat{v}}: \begin{cases} \delta w^v_{1(k+1)} = T(\phi^v_2(w^v_1) + w^v_2) + w^v_1, & \text{(23a)}\\ \delta w^v_{2(k+1)} = T(\phi^v_1(w^v_1) + e_v) + w^v_2, & \text{(23b)}\\ \delta \hat{v}_{(k+1)} = T\left(\phi^v_0(w^v_1) + \tilde{f}_e - \dfrac{\hat{f}_r + \hat{f}_k + f_u}{m}\right), & \text{(23c)}\\ \delta \tilde{f}_{e(k+1)} = T\phi^v_0(w^v_1), & \text{(23d)} \end{cases}$$

where the notation $\delta a_{(k+1)} = a_{(k+1)} - a_{(k)}$ is employed, and the superindex v is used to indicate the functions belong to the velocity HSMO $\Sigma_{\hat{v}}$. On the other hand, the HSMO for position estimation is a SM zero-order filter, given by:

$$\Sigma_{\hat{x}}: \begin{cases} \delta w^x_{1(k+1)} = T(\phi^x_2(w^x_1) + w^x_2) + w^x_1, & \text{(24a)}\\ \delta w^x_{2(k+1)} = T(\phi^x_1(w^x_1) + e_x) + w^x_2, & \text{(24b)}\\ \delta \hat{x}_{(k+1)} = T\phi^x_0(w^x_1). & \text{(24c)} \end{cases}$$

In Table I, the remainder of the parameters, for the design of $\Sigma_{\hat{v}}$ and $\Sigma_{\hat{x}}$, are presented.

To assess the accuracy of the proposed HSMO, the error is calculated. Then, the simulation results are mainly focused on the results of the reconstruction of f_e. These results are compared with a Kalman filter unknown input observer, designed to obtain an accurate f_e estimate [4].

TABLE I
NOMINAL SYSTEM MODEL, UNKNOWN INPUT OBSERVER PARAMETERS AND NORMALISED ROOT MEAN SQUARE ACCURACY

WEC model		
$\mathcal{M} = 6.8 \times 10^{-6}$	$k_z = 5.57 \times 10^5$	$n_r = 7$
Parameter	**x-HSMO**	**v-HSMO**
L_i	5 $[m/s^2]$	6 $[m/s^3]$
λ_0	1.1	1.1
λ_1	3.06	2.12
λ_2	4.16	2
λ_3	3	N/A
ε_0	$\mathcal{N}(0, 9 \cdot 10^{-6})$	$\mathcal{N}(0, 2.5 \cdot 10^{-5})$
ε_1	0	Equation (29)
State estimation precision		
NRMSA	**HSMO**	**KFHO - R** $= 10^{-7}$
e_x	99.95	99.41
e_v	99.93	99.94
$\mathbf{H}e_{\mathbf{x}_r}$	100	99.99

A. Kalman filter with harmonic oscillator

In this subsection, a KF with a harmonic oscillator (KFHO) is presented. This estimator has proven to have a good response to noisy signals [4]. The KFHO is, in essence, a linear observer with extended dynamics, which are

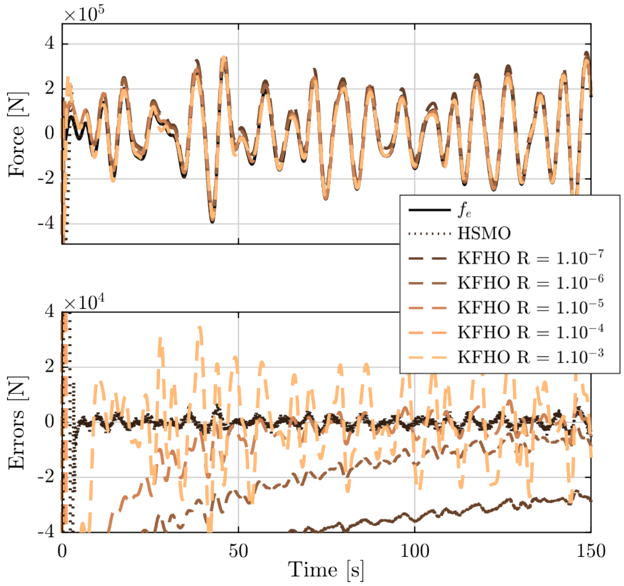

Figure 1. Excitation force and force estimates when a difference in the initial conditions between the model and the plant is considered. (a) Excitation force and force estimates for each observer. (b) Estimation errors.

considered to model the temporal variation of the excitation force, which is assumed to be of the form:

$$\hat{f}_e^{KF} = \sum_{i=1}^{n_q} A_i \sin(\omega_i t + \theta_i), \tag{25}$$

where $n_q \in \mathbb{N}$. Depending on the number of considered frequencies n_q, the KFHO uses $2n_q$ extra states in the system model (8) to design the observer. Succinctly, the KFHO employs the nominal system matrices from (8) and is extended as:

$$\Sigma_K \begin{cases} \dot{\boldsymbol{x}}_e = \begin{bmatrix} \mathbf{A} & \mathbf{BC}_f \\ \mathbf{0} & \mathbf{A}_f \end{bmatrix} \boldsymbol{x}_e + \mathbf{L}_k(y - \hat{y}), & \text{(26a)} \\ \hat{f}_e = \begin{bmatrix} \mathbf{0} & \mathbf{C}_f \end{bmatrix} \boldsymbol{x}_e, & \text{(26b)} \\ \hat{y} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \boldsymbol{x}_e, & \text{(26c)} \end{cases}$$

with $\mathbf{L}_k$ being the infinite horizon Kalman gain, which depends on the measurement and process noise covariance matrices (see Subsection IV-B), and:

$$\mathbf{A}_f = \bigoplus_{i=1}^{n_q} \begin{bmatrix} 0 & \omega_i \\ \omega_i & 0 \end{bmatrix}, \; \mathbf{C}_f = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \end{bmatrix}. \tag{27}$$

For further details, the reader may refer to [4], [5], [23].

B. Process and measurement covariance matrix selection

One of the main advantages of the Kalman filter is that it can consider uncertainties in the model, while it also incorporates information on the sea spectrum. In case the system is a precisely identified model of the plant, the process noise covariance matrix associated with the WEC model is typically:

$$\mathbf{Q} = \mathbf{0}_n \oplus q\mathbf{I}_{2\,n_q}, \tag{28}$$

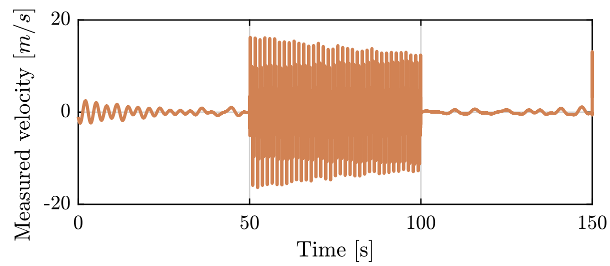

Figure 2. Measured velocity $\boldsymbol{v}$ with gaussian noise and noise from (29) between 50s - 100s.

with $q \in \mathbb{R}$ being large. Regarding sensor characterisation, the associated covariance matrix $\mathbf{R}$ must be selected considering the sensor characteristics and expected measurement noise. In combination with $\mathbf{Q}$, the selection of $\mathbf{R}$ has a major impact on the obtained results. This is illustrated in Subsection IV-C.

C. Estimator comparison

To evaluate the robustness of each observer, measurement Gaussian noise ε_0, in accordance with [4] is considered. This is included to represent sensor measurements with spurious components within the signal bandwidth (see Table I).

In Figure 1, the $\hat{f}_e$ estimates obtained with the KFHO, employing different $\mathbf{R}$ values, are presented and compared with the HSMO, when there is 10% error in the initial conditions. Regarding the KFHO estimates, it can be noted that, with smaller $\mathbf{R}$, the convergence is slower, but $\hat{f}_e$ presents minor amplitude oscillations when compared with the cases with larger $\mathbf{R}$. This is because, when a perfect model is assumed, and $\mathbf{R} \to 0$, the KF-based observer tends to a high-gain observer performing a plant inversion which is, conceptually, the procedure performed by the HSMO [22].

The normalised root mean square accuracy (NRMSA) [4] of the HSMO and the KFHO for the estimate of the states, when the noise $\varepsilon = \varepsilon_0$, is presented in Table I. It is worth noting that the results obtained with the HSMO depend only on the appropriate selection of L which, together with the unbounded noise rejection capabilities of the SM filter, provide the HSMO with considerable design flexibility. To illustrate the latter aspect, the following scenario (focused on the reconstruction of f_e) is proposed; the noise is assumed to be $\varepsilon = \varepsilon_0 + \varepsilon_1$, satisfying Assumption 1, with ε_1 of the filtering order 2, specified as:

$$\varepsilon_1(t) = k_e \frac{2}{3} \frac{d^2}{dt^2} \left(\lfloor \cos(1000t) \rceil^{3/2} \right) =$$
$$= k_e \left(\frac{\sin^2(ft) \lfloor \cos(ft) \rceil^{\frac{-1}{2}}}{2} - \cos^2(ft) \lfloor \cos(ft) \rceil^{\frac{-1}{2}} \right), \tag{29}$$

with $k_e = 7, 5.10^{-6}$, $f = 1000$ [Hz], and included in the interval between $50s$ and $100s$ (see Figure 2). On the one hand, in Figure 3 a) and b), the KFHO assumes $\mathbf{R} = 10^{-3}$ to deal with the noise from (29), at the expense of reducing the accuracy when the noise (29) is absent. On the other hand, assuming $\mathbf{R} = 10^{-7}$ (in Figure 3 c) and d)) to increase the f_e estimate precision, in the intervals when (29) is not

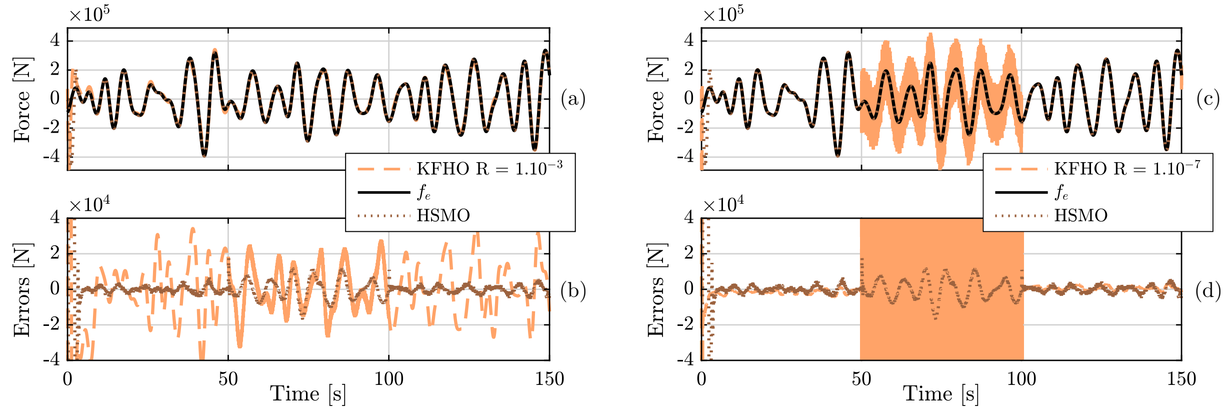

Figure 3. Estimation with initial conditions equal to zero. Figures (a) and (c) show the excitation force and the estimates, figures (b) and (d) show the error in each estimation. For (a) and (b) $\mathbf{R} = 10^{-3}$. For (c) and (d) $\mathbf{R} = 10^{-7}$.

zero, a considerable degradation in $\hat{f}_e$ appears. As a result, although it is well known that noise covariance matrices have a significant impact on the obtained estimates, the proposed HSMO is capable of matching the results obtained with the KFHO without changes in the observer structure, such as adaptive estimation of the covariance matrix [24].

V. Conclusions

The present paper develops a novel sliding mode-based unknown input observer. The proposal is a homogeneous structure, which guarantees finite time convergence of the estimation error associated with the homogeneous part of the observer, with asymptotically optimal performance. The latter means that, in the absence of measurement noise, the estimation is exact. In addition, the advantages of the proposal include simplicity, provided a Lipschitz bound is the only essential design parameter, the possibility to reject unbounded noises, and a low computational burden for its real-time implementation.

The accuracy of the proposed HSMO is analysed by comparing the estimation results against a KF with a harmonic oscillator, a typical observer utilised in the wave energy field. The results show that this paper proposal is capable of matching the results obtained with the KFHO under a Gaussian measurement noise assumption and improving the results with unbounded noises, without the requirement of covariance matrix adaptation.

References

[1] J. V. Ringwood, S. Zhan, and N. Faedo, "Empowering wave energy with control technology: Possibilities and pitfalls," *Ann. Rev. Control*, Apr. 2023.

[2] J. Falnes and A. Kurniawan, *Ocean Waves And Oscillating Systems: Linear Interactions Including Wave-Energy Extraction*. Cambridge University Press, 2020, vol. 8.

[3] N. Faedo, S. Olaya, and J. V. Ringwood, "Optimal control, MPC and MPC-like algorithms for wave energy systems: An overview," *IFAC J. Syst. Control*, vol. 1, pp. 37–56, Sep. 2017.

[4] Y. Peña-Sanchez, C. Windt, J. Davidson, and J. V. Ringwood, "A critical comparison of excitation force estimators for wave-energy devices," *IEEE Trans. Control Syst. Technol.*, Nov. 2019.

[5] B. A. Ling and B. A. Batten, "Real time estimation and prediction of wave excitation forces on a heaving body," in *ASME International Conference on Offshore Mechanics and Arctic Engineering, St Johns*, vol. 56574, May 2015.

[6] B. Guo, R. J. Patton, S. Jin, and J. Lan, "Numerical and experimental studies of excitation force approximation for wave energy conversion," *Renewable Energy*, vol. 125, pp. 877–889, Sep. 2018.

[7] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Trans. ASME, J. Basic Eng.*, vol. 82D, pp. 35–45, Mar. 1960.

[8] A. Levant and M. Livne, "Robust exact filtering differentiators," *Eur. J. Control*, vol. 55, pp. 33–44, Sep. 2019.

[9] A. F. Filippov, *Differential Equations with Discontinuous Righthand Sides*, F. M. Arscott, Ed. Springer Netherlands, 1988.

[10] J. A. Moreno, "A linear framework for the robust stability analysis of a generalized super-twisting algorithm," in *2009 6th Int. Conf. Elect. Eng. Comput. Sci. Autom. Control (CCE)*. IEEE, Jan. 2009, pp. 1–6.

[11] J. A. Moreno, "On strict Lyapunov functions for some non-homogeneous super-twisting algorithms," *J. Franklin Inst.*, vol. 351, no. 4, pp. 1902–1919, Apr. 2014.

[12] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control*, vol. 76, no. 9-10, jan 2003.

[13] J.-P. Barbot, A. Levant, M. Livne, and D. Lunz, "Discrete differentiators based on sliding modes," *Automatica*, vol. 112, Feb. 2020.

[14] W. E. Cummins, "The impulse response function and ship motions," Schiffstechnik, Tech. Rep., 1962.

[15] T. Pérez and T. I. Fossen, "Time-vs. frequency-domain identification of parametric radiation force models for marine structures at zero speed," *Model. Identif. Control*, vol. 29, no. 1, pp. 1–19, 2008.

[16] A. Levant, "Robust exact differentiation via sliding mode technique," *Automatica*, vol. 34, no. 3, pp. 379–384, mar 1998.

[17] A. Levant, "Homogeneity approach to high-order sliding mode design," *Automatica*, vol. 41, no. 5, pp. 823–830, may 2005.

[18] A. S. Poznyak, "Stochastic output noise effects in sliding mode state estimation," *Int. J. Control*, vol. 76, no. 9-10, pp. 986–999, jan 2003.

[19] T. Floquet, C. Edwards, and S. K. Spurgeon, "On sliding mode observers for systems with unknown inputs," *Int. Journal of Adaptive Control and Signal Proc.*, vol. 21, no. 8–9, pp. 638–656, May 2007.

[20] R. Seeber and S. Koch, "Structural conditions for chattering avoidance in implicitly discretized sliding mode differentiators," *IEEE Control Syst. Lett.*, vol. 7, pp. 2065–2070, Jun. 2023.

[21] F. Ardhuin, J. E. Stopa, B. Chapron, F. Collard, R. Husson, R. E. Jensen, J. Johannessen, A. Mouche, M. Passaro, G. D. Quartly, V. Swail, and I. Young, "Observing sea states," *Frontiers in Marine Science*, vol. 6, Apr. 2019.

[22] F. D. Mosquera, P. Fornaro, P. F. Puleston, C. A. Evangelista, and J. V. Ringwood, "A sliding mode-based tracking observer for excitation force estimation in wave energy systems," in *Conf. Control Technology Applications (CCTA)*. Newcastle, UK: IEEE, Aug. 2024.

[23] D. García-Violini, Y. Peña-Sanchez, N. Faedo, C. Windt, F. Ferri, and J. V. Ringwood, "Experimental implementation and validation of a broadband LTI energy-maximizing control strategy for the Wavestar device," *IEEE Trans. Control Syst. Technol.*, vol. 29, no. 6, Jan. 2021.

[24] L. Zhang, D. Sidoti, A. Bienkowski, K. R. Pattipati, Y. Bar-Shalom, and D. L. Kleinman, "On the identification of noise covariances and adaptive Kalman filtering: A new look at a 50 year-old problem," *IEEE Access*, vol. 8, pp. 59 362–59 388, Mar. 2020.